near the front. The maximum displacement of the latter corresponds to the neighborhood of the pressure singularity.

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DISINTEGRATION OF AN ARBITRARY DISCONTINUITY IN A PERFECTLY CONDUCTING MAGNETIZABLE INCOMPRESSIBLE MEDIUM

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V. A. NALETOVA and G. A. SHAPOSHNIKOVA
(Moscow)

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Centered waves and strong discontinuities in a perfectly conducting magnetizable incompressible medium are investigated. It is shown that in shock waves in such medium the magnetic field tangential to the discontinuity plane and the magnetic induction increase, and the magnetic permeability decreases. In centered waves the tangential magnetic field and magnetic induction decrease. The problem of disintegration of an arbitrary discontinuity in a magnetizable perfectly conducting incompressible medium is solved by constructing diagrams in the plane of components of the tangential velocity initial shock. The diagrams make possible the determination of the combination of waves and discontinuities formed at disintegration.

Let at the initial instant of time $t = 0$ parameters $\mathbf{B}_\tau$, $\mathbf{H}_\tau$, $\mathbf{v}_\tau$, and T become discontinuous in the plane $x = 0$.

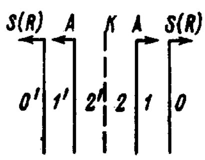

Fig. 1

If the laws of conservation are not satisfied at the discontinuity, the latter cannot exist, and it is necessary to determine the motion of medium at the following instants of time. The self-similarity of the problem implies that the motion must consist of a combination of shock waves S, centered waves R, rotational Alfven discontinuities A and a contact discontinuity K. It will be shown below that the propagation velocity of such waves (with the model considered here) is such, that two waves may propagate in each direction from the contact discontinuity that separates them (Fig. 1). The gasdynamic problem of the disintegration of an arbitrary discontinuity in a perfect gas was solved in [1, 2], for a medium with an arbitrary equation of state it was solved in [3], and for combustible mixtures in [4]. The magnetohydrodynamic problem of disintegration of an arbitrary discontinuity was solved in [5].

We denote the parameters that define the medium at the initial instant of time by the subscript zero. Parameters of the medium that lies at the initial instant of time to the left of the discontinuity plane and subsequently to the left of the contact discontinuity plane are denoted by a prime. Parameters of the medium lying to the right of corresponding surfaces are denoted by letters without primes, and parameters of the medium behind the first wave moving either right or left are denoted by the numeral 1, while those behind the A -discontinuity by numeral 2.

1. Basic equations. The system of equations that defines continuous flows of a magnetizable perfectly conducting fluid may be written as [6-8]

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{v} = 0, \quad p = p_0 + \frac{1}{4\pi}\int_0^H \left(\mu - \rho\frac{\partial \mu}{\partial \rho}\right) H dH \tag{1.1}$$

$$\rho\frac{d\mathbf{v}}{dt} = \nabla p + \frac{1}{4\pi}\operatorname{rot}\mathbf{H}\times\mathbf{B} + \frac{\mathbf{B}\nabla\mathbf{H}}{4\pi}$$

$$\frac{\partial}{\partial t}\left(\rho\frac{v^2}{2} + \rho U_m\right) = -\operatorname{div}\left\{\rho\mathbf{v}\left(\frac{v^2}{2} + \frac{p}{\rho} + U_m - \frac{HB}{4\pi\rho}\right) + \frac{c}{4\pi}\mathbf{E}\times\mathbf{H}\right\}$$

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v}\nabla)\mathbf{B} = (\mathbf{B}\Delta)\mathbf{V}, \quad c\mathbf{E} = -\mathbf{v}\times\mathbf{B}, \quad \mathbf{B} = \mu\mathbf{H} = \mathbf{H} + 4\pi\mathbf{M}$$

$$U_m = U_{m0} + \frac{1}{4\pi\rho}\mathbf{BH} + \frac{1}{4\pi}\int_0^H \frac{1}{\rho}\left(T\frac{\partial\mu}{\partial T} - \mu\right) H dH$$

where the notation is the same as in [8]. It is possible to obtain from (1. 1) in a system of coordinates in which the discontinuity is at rest the relationships valid at the discontinuity surfaces [7]

$$\{\rho v_n\} = 0, \quad \{\rho v_n^2 + p - B_n H_n / 4\pi\} = 0 \tag{1.2}$$

$$\{B_n\} = 0, \quad B_n\{\mathbf{v}_\tau\} = \{v_n \mathbf{B}_\tau\}, \quad \rho v_n\{\mathbf{v}_\tau\} = B_n\{\mathbf{H}_\tau\} / 4\pi$$

$$\left\{\rho v_n\left(\frac{v^2}{2} + \frac{p}{\rho} + U_m - \frac{B_n H_n}{4\pi\rho}\right) - \frac{B_n}{4\pi} H_\tau v_\tau\right\} = 0$$

where subscripts τ and n denote vector components tangent and normal to the discontinuity surface $\{F\} = F_1 - F_0$, where F_0 and F_1 represent values of F to the left and right of the discontinuity, respectively. The solution of system (1. 2) must satisfy the condition of nondiminution of entropy

$$\left\{S_{H=0} + \frac{1}{4\pi\rho}\int_0^H \left(\frac{\partial\mu}{\partial T}\right)_{\rho, H} H dH\right\} \geqslant 0 \tag{1.3}$$

where $S_{\mathbf{H}=0}$ is the entropy of the medium in a zero magnetic field. Subsequently we consider an incompressible saturated magnetic material which satisfies the following equations of state :

$$\rho = \text{const}, \quad U_{m0} = cT, \quad M = K(\Theta - T) \tag{1.4}$$

where Θ is the Curie temperature and $K = \text{const}$.

2. Centered waves in an incompressible perfectly conducting magnetizable medium. We seek a solution of system (1.1) of the form

$$\mathbf{v}_\tau = \mathbf{v}_\tau\left(\frac{x}{t}\right), \quad T = T\left(\frac{x}{t}\right), \quad p = p\left(\frac{x}{t}\right) \tag{2.1}$$

$$\mathbf{B} = \mathbf{B}_n\left[\mathbf{l} + \mathbf{b}\left(\frac{x}{t}\right)\right], \quad \mathbf{v}_\tau \mathbf{l} = \mathbf{b}\mathbf{l} = 0$$

where l is a unit vector directed along the x-axis. From system (1.1) we obtain for the definition of propagation of simple centered waves the equations

$$p - \frac{B_n H_n}{4\pi} = \text{const}, \quad \tau(b) = \tau_0 \exp\{m\beta(\sqrt{1+b^2} - \sqrt{1+b_0^2}) + m\tau(b) - m\tau_0\} \tag{2.2}$$

$$\frac{\partial \mathbf{v}_\tau}{\partial \xi} = -(\xi - v_n)\frac{\partial \mathbf{b}}{\partial \xi}, \quad \frac{\partial \mathbf{v}_\tau}{\partial \xi} = -\frac{a_0^2}{\xi - v_n}\frac{\partial}{\partial \xi}[\mathbf{b}f(b)]$$

$$f(b) = 1 - \frac{4\pi M(\tau, b)}{B_n\sqrt{1+b^2}}, \quad \xi = \frac{x}{t}, \quad a_0^2 = \frac{B_0^2}{4\pi\rho}$$

$$\tau = \frac{T}{\Theta}, \quad \tau_0 = \frac{T_0}{\Theta}, \quad b_0 = \frac{|B_{\tau 0}|}{B_n}, \quad \beta = \frac{B_n^2}{4\pi\rho c\Theta}, \quad m = \frac{4\pi K\Theta}{B_n}$$

where $\mathbf{B}_{\tau 0}$ and T_0 are the magnetic induction and the medium temperature in the unperturbed state. From the third and fourth equations of system (2.2) we obtain

$$(\xi_1 - v_n)^2 = a_0^2 f(b), \quad (\xi_2 - v_n)^2 = a_0^2(f + b_y f_{by}' + b_z f_{bz}') \tag{2.3}$$

Since in a centered wave propagating at velocity ξ_1 the absolute value of the magnetic induction vector does not change, hence $\xi_1 = \text{const}$. Such centered wave degenerates into a weak discontinuity that propagates at the Alfven velocity. In a wave that propagates relative to the medium at velocity $a = \xi_2 - v_n$ the direction of vector $\mathbf{B}$ does not vary, the wave may be considered to be plane. The propagation velocity of a wave can be defined by

$$a^2(b) = a_A^2 + \frac{B_n^2 b^2}{4\pi\rho(1+b^2)}\left[\frac{4\pi M}{B_n(1+b^2)^{1/2}} + \frac{m\beta\tau}{1 - m\beta\tau}\right] \tag{2.4}$$

$$a_A^2 = \frac{B_n^2}{4\pi\rho\mu}$$

The tangential velocity distribution in such wave is linked with vector b by the relation

$$\mathbf{v}_\tau = \mathbf{v}_{\tau 0} \mp \frac{\mathbf{b}}{b}\int_{b_0}^{b} a(b)\,db \tag{2.5}$$

where $v_{\tau 0}$ is the velocity of unperturbed medium. The propagation velocity of the simple wave $a(b)$ and the relation between parameter $\mathbf{v}_\tau$ and $\mathbf{b}$ (2.5) in the wave were determined in [9].

Let us consider the variation of quantities in a plane centered wave (2.5) where $\partial b / \partial \xi = \xi / (a\partial a / \partial b)$, which implies that sign $(\partial b / \partial \xi) = \pm$ sign $(\partial a / \partial b)$.

The upper and lower signs in this formula and in (2.5) correspond to centered waves moving to the right and left, respectively.

When $(\partial a / \partial b) > 0$ only centered waves for which the absolute value of magnetic induction vector decreases are possible, while for $(\partial a / \partial b) < 0$ the absolute value of that vector increases. In the case of $\beta \ll 1$, $m \lesssim 1$

$$\frac{\partial a}{\partial b} = \frac{B_n^2}{4\pi\rho a(b)}\left[\frac{3(1-\tau(b))b}{(1+b^2)^{5/2}} + \frac{3\tau_0\beta m b}{(1+b^2)^2}\right] > 0 \tag{2.6}$$

only centered waves for which the absolute value of the magnetic induction vector decreases are possible. It follows from the second of Eqs. (2.2) that for $\beta \ll 1$ and $m \lesssim 1$ the initial temperature distribution in waves remains unchanged, and the magnetic induction vector is expressed in terms of the magnetic field by

$$\mathbf{B} = \mathbf{H} + \frac{4\pi K(\Theta - T)}{|\mathbf{H}|}\mathbf{H} \tag{2.7}$$

Let us compare the propagation velocity of weak perturbations a with the Alfven velocity a_A. It is seen from (2.4) that $a(b) < a_A$ when $m\beta\tau > 1$ and $a(b) > a_A$ when $m\beta\tau < 1$ When $\beta \ll 1$ and $m \lesssim 1$ the velocity of small perturbations $a(b)$ is higher than the Alfven velocity

$$a(b) > a_A \tag{2.8}$$

By expressing the magnetic induction vector $\mathbf{B}_\tau$ in terms of the magnetic field vector $\mathbf{H}$ as in formula (2.7), the relationship at centered waves (2.5) can be written in the form

$$\mathbf{v}_{\tau 1} = \mathbf{v}_{\tau 0} \mp \chi(|\mathbf{H}_{\tau 1}|, |\mathbf{H}_{\tau 0}|)\frac{\mathbf{H}_{\tau 0}}{|\mathbf{H}_{\tau 0}|}, \quad \frac{\partial \chi}{\partial H_{\tau 1}} > 0 \tag{2.9}$$

3. <u>Shock waves in a perfectly conducting magnetizable medium.</u>

The relationships at a shock wave in incompressible fluid were considered in [9]. It was assumed there that in a zero magnetic field the enthalpy is proportional to the temperature $w_0 = cT$. Here we use the relationship $U_{m0} = cT$, where U_{m0} is the internal energy of fluid in a zero magnetic field. This explains why the results of shock wave investigation in the present work differ from those in [9]. With allowance for (1.4) system (1.2) can be transformed to

$$\alpha^2\left\{b - \frac{m'b}{\sqrt{1+b^2}}\right\} = \{b\}, \ \{v_\tau\} = v_n\{b\}, \ \frac{8\pi}{B_n^2}\{p\} = -\left\{\frac{2m'}{\sqrt{1+b^2}}\right\} \tag{3.1}$$

$$\frac{2}{\beta}\{\tau\} = \left\{\alpha^2 b^2\left(1 - \frac{m'}{\sqrt{1+b^2}}\right)^2 + 2m(\sqrt{1+b^2} - m') + \frac{2m'}{\sqrt{1+b^2}} + (\sqrt{1+b^2} - m')^2 - 2b^2\left(1 - \frac{m'}{\sqrt{1+b^2}}\right)\right\}, \quad \alpha^2 = \frac{B_n^2}{4\pi\rho v_n^2},$$

$m' = m(1-\tau)$

$(\mathbf{B}_{\tau 0} \parallel \mathbf{B}_{\tau 1} \parallel \{\mathbf{v}_\tau\})$

The condition for the nondiminution of entropy (1.3) is of the form

$$\{\tau\} \geqslant \tau_0\left[\exp\left\{m\beta\sqrt{1+b_0^2}\left(\frac{1-\varphi}{\varphi} + \frac{m\{\tau\}}{\sqrt{1+b_0^2}}\right)\right\} - 1\right] \tag{3.2}$$

$$\varphi = \varphi(\xi, b_0) = \sqrt{\frac{1 + b_0^2}{1 + b_0^2\xi^2}}, \quad \xi = \frac{b_1}{b_0}$$

The fourth of Eqs. (3. 1) is a quadratic equation with respect to the temperature jump $\{\tau\}$. When $\beta \ll 1$ and $m \lesssim 1$ one of the roots $\{\tau\}_1 \sim 1 / \beta$ has no physical meaning, and the second can be written in the form

$$\{\tau\} = \beta\left(\frac{m\sqrt{1+b_0^2}\,(1-\varphi)}{\varphi} - \frac{b_0^2(\xi-1)\,m_0(\xi\varphi-1)}{2}\right), \quad m_0 = \frac{m\,(1-\tau_0)}{\sqrt{1+b_0^2}} \tag{3.3}$$

It follows from this that in such shocks the temperature of medium does not change. For $\beta \ll 1$ and $m \lesssim 1$ condition (3. 2) is transformed to

$$\{\tau\} \geqslant m\beta\tau_0\sqrt{1 + b_0^2}\,(1 - \varphi)\varphi^{-1} \tag{3.4}$$

With allowance for (3. 3) and (3. 4) we have

$$(1 + b_0^2)\frac{1-\varphi}{\varphi} \geqslant \frac{b_0^2}{2}(\xi - 1)(\xi\varphi + 1) \tag{3.5}$$

Condition (3. 5) implies that $b_1 \geqslant b_0$. Hence shock waves which increase the magnetic induction and magnetic field vectors are possible. The magnetic permeability is in this case diminished.

From the first of Eqs. (3. 1) and Eq. (3. 3) follows

$$\gamma^2 = (1 - m_0)^{-1}\left(1 - m_0\frac{\xi\varphi - 1}{\xi - 1}\right) \tag{3.6}$$

$$m_0 = \frac{m\,(1-\tau_0)}{\sqrt{1+b_0^2}}, \quad \gamma = \frac{v_n}{a_{A0}} = \alpha^{-1}(1 - m_0)^{-1/2}$$

Using the inequalities $m_0 < 1$ and $0 \leqslant (\xi\varphi - 1) / (\xi - 1) \leqslant 1$ (the latter inequality is valid for any $\xi > 0$ and $b_0 > 0$), from (3. 6) we obtain $1 \leqslant \gamma^2 < (1 - m_0)^{-1}$). The last formula shows that the considered shocks can only propagate at velocities higher than the Alfven velocity. The second of Eqs. (3. 2) determines the relation between tangential components of the magnetic induction vector and the velocity of medium

$$\mathbf{v}_{\tau 1} - \mathbf{v}_{\tau 0} = \mp a_{A0}\ |\gamma(b_1, b_0)|\ (\mathbf{b}_1 - \mathbf{b}_0) \tag{3.7}$$

where the upper and lower signs relate to waves propagating to the right and left, respectively. Using formula (2. 7) for expressing the magnetic induction vector in terms of the magnetic field vector we obtain

$$\mathbf{v}_{\tau 1} - \mathbf{v}_{\tau 0} = \mp \Psi(|\mathbf{H}_{\tau 1}|, |\mathbf{H}_{\tau 0}|)(\mathbf{H}_{\tau 1} - \mathbf{H}_{\tau 0}), \quad \Psi > 0 \tag{3.8}$$

Using the method proposed in [11, 12] for investigating the interaction between a shock wave and small perturbations propagating at velocities[10] $a_A = B_n / \sqrt{4\pi\rho\mu}$, a and a_l (a_l is the velocity of entropy wave propagation), for the condition of shock wave evolution we obtain

$$v_n > \max\{a_{A0}, a_{A1}\}, \quad a_0 \leqslant v_n \leqslant a_1 \tag{3.9}$$

$$\text{or} \quad v_n < \min\{a_{A0}, a_{A1}\}, \quad a_0 \leqslant v_n \leqslant a_1$$

when $\beta \ll 1$ and $m \lesssim 1$ Eq. (2. 4) and the second of Eqs. (2. 2) may be written in the form

$$a^2 = \frac{B_n^2}{4\pi\rho}\left(1 - \frac{m\,(1-\tau)}{(1+b^2)^{3/2}} + \frac{b^2m^2\beta\tau_0}{1+b^2}\right) \tag{3.10}$$

$$\tau = \tau_0 \, [1 + m\beta \, (\sqrt{1 + b^2} + m\tau_0)] \tag{3.11}$$

which are accurate to within infinitely smalls of order β^2. The condition $a_1 > a_0$ with Eqs. (3. 10) and (3. 11) imply that

$$\frac{-1 + \tau_0 + \tau_0^2 \beta m^2}{(1 + b_1^2)^{3/2}} \geqslant \frac{-1 + \tau_0 + \tau_0^2 \beta m^2}{(1 + b_0^2)^{3/2}} \tag{3.12}$$

When $\beta \ll 1$ the inequality $-1 + \tau_0 + \tau^2_0 \beta m^2 < 0$ is always satisfied, and from inequality (3. 12) follows that $b_1 \geqslant b_0$. Conditions (3. 9) are satisfied when the inequality $b_1 \geqslant b_0$ is satisfied. Henceforth the case of $\beta \ll 1$ and $m \lesssim 1$ will be considered.

All calculations and estimates presented in Sect. 3 are valid for $\xi \ll \min \{\tau_0, 1 - \tau_0\} / m\beta b_0$. This inequality imposes a restriction on the magnetic induction vector behind the shock wave.

4. **Conditions at rotational and contact discontinuities in perfectly conducting incompressible magnetisable media.** At a rotational discontinuity only $\mathbf{H}_\tau$ and $\mathbf{v}_\tau$, i.e. the tangential components of the magnetic field and velocity respectively are discontinuous, while the magnetic field remains unchanged.

The variation of tangential velocity components and of the field is defined by the relation

$$\mathbf{v}_{\tau 2} - \mathbf{v}_{\tau 1} = \mp \frac{a_1 \mu}{B_n} (\mathbf{H}_{\tau 2} - \mathbf{H}_{\tau 1}) \tag{4.1}$$

The upper and lower signs correspond to waves propagating to the right and left, respectively. At the contact discontinuity

$$\mathbf{H}_{\tau 2} = \mathbf{H}'_{\tau 2}, \quad \mathbf{v}_{\tau 2} = \mathbf{v}'_{\tau 2} \tag{4.2}$$

The temperature and pressure may be discontinuous.

5. **Disintegration of an arbitrary discontinuity (the plane case).** Let us consider the plane problem of disintegration of an arbitrary discontinuity $\mathbf{H}_{\tau 0} \parallel \mathrm{H}'_{\tau 0} \parallel \mathbf{v}_{\tau 0} \parallel \mathbf{v}_{\tau 0}'$. Below in this Section we omit the vector symbol and the subscript τ. The condition of evolution (3. 9) and the second of equalities (2. 7) imply that a shock wave S or centered wave R propagate to both sides of the contact discontinuity, which are followed by Alfven discontinuities Such flow is diagrammatically shown in Fig. 1.

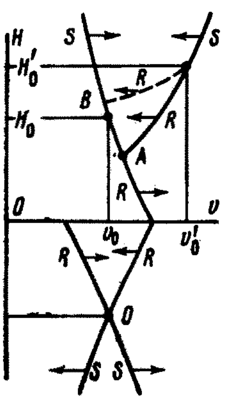

Fig. 2

Let us consider in the vH-plane the curves defined by Eqs. (3, 8) and (2. 9) which in Fig. 2 are, respectively, denoted by letters S and R These curves issue from points with coordinates v_0, H_0 and v_0', H_0' and correspond to waves propagating to the right through the medium with parameters v_0, H_0, and T_0, and to the left through the medium with parameters v_0', H_0', and T_0' respectively. The curves shown emanating from point O_1 in the lower half-plane in Fig. 2 correspond to waves S and R which move to the right and left through the medium in which the tangential component of the magnetic field is negative. The tangential component of the magnetic field may decrease to zero in the case of centered waves, while in shock waves it can increase from zero to ∞.

In Fig. 2 is shown the case when $H_0 < H_0'$ and $v_0 < v_0'$, and the line that corresponds to the centered wave moving to the left through the medium with parameters v_0', H_0' and T_0' intersects the line that corresponds to the centered wave moving to the right through the medium with parameters of state denoted by subscript zero (these two curves are shown in Fig. 2 by solid lines).

The intersection point A at coordinates v_1H_1 corresponds to the state behind the fronts of centered waves moving in opposite directions away from the initial discon - tinuity. This represents the particular case of disintegration of an arbitrary discon - tinuity into two centered waves : RKR. A contact discontinuity may exist between these waves at which the temperature and pressure are discontinuous. The case in which wave R moving through the medium with parameters v_0', H_0', and T_0' does not intersect wave R but intersects wave S which moves through the medium with parameters v_0, H_0, and T_0 is shown in Fig. 2 by a dash line. The point of intersection is denoted by B whose coordinates correspond to the state behind the fronts of the shock wave running to the right and the centered wave running to the left of the initial discontinuity.

A contact discontinuity, at which the temperature and pressure are discontinuous, exists between such waves. We have the particular case of disintegration of an arb - itrary discontinuity into a shock and a centered wave : RKS. The regions of existence of combinations RKR and RKS are separated by the line emanating from point v_0H_0 and defined by the equation $v_0 = v + \chi (H_0, H)$ (Fig. 3). This line corresponds to a centered wave that propagates to the left and changes the magnetic field and velocity from v_0' and H_0' to v_0H_0. Such line represents the solution of the problem of disintegration of an arbitrary discontinuity into a centered wave moving to the left and a contact discontinuity RK.

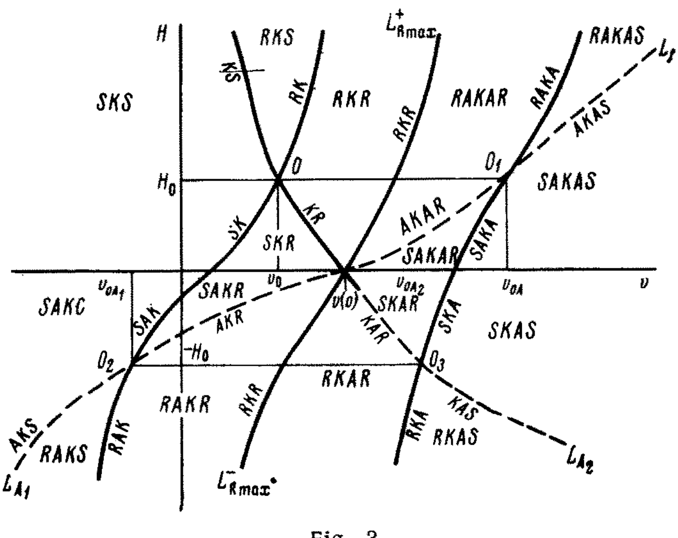

Fig. 3

The boundaries of all regions in which one or the another variant of disintegration of an arbitrary discontinuity into two waves moving in opposite directions is realized can be similarly constructed in the vH-plane for fixed v_0, H_0, and T_0.

Sixteen regions are shown in Fig. 3. in the vH -plane. Each of these corresponds to a particular combination of two waves moving to the right and left, of A - discontinuities which follow these and rotate the magnetic field by $180°$, and of a contact discontinuity lying between these.

Let us consider the regions of parameters denoted by subscript zero at which the disintegration of an arbitrary discontinuity is not accompanied by A-discontinuities. Four combinations are possible: RKR, SKS, RKS and SKR. The regions of these configurations lie in the upper half-plane to the left of line $L_{R\max}^{+}$ which is defined by the equation

$$v_0 - \chi(0, H_0) = v + \chi(0, H) \tag{5.1}$$

This line represents the relation between quantities in the centered wave which propagates to the left; it makes it possible to reach $v(0)$, where 0 is the point of intersection of the line, which corresponds to the centered wave moving through the medium with parameters v_0, H_0, and T_0, with the axis $H = 0$.

The line issuing from point v_0, H_0 corresponds to combinations RK, KS, SK, and KR (Fig. 3). The equations of these lines are

$$\begin{aligned}
RK:&\quad v_0 = v + \chi(H_0, H), \quad v > v_0 \\
KS:&\quad v = v_0 - \Psi(H, H_0)(H - H_0), \quad v < v_0 \\
SK:&\quad v_0 = v + \Psi(H_0, H)(H_0 - H), \quad v < v_0 \\
KR:&\quad v = v_0 - \chi(H, H_0), \quad v > v_0
\end{aligned} \tag{5.2}$$

To the right of line $L_{R\max}^{+}$ and in the lower half-plane lie regions of parameters $v_0'H_0'$ in which Alfven discontinuities that turn the magnetic field vector by $180°$ must necessarily accompany the disintegration of an arbitrary discontinuity.

Let us consider all possible states O' in which the disintegration of an arbitrary discontinuity is accompanied by two A-discontinuities that propagate in opposite directions from the initial discontinuity behind the S- or K-waves. Each of these A-discontinuities turn the magnetic field vector by $180°$. The points that correspond to such states lie in the upper part of the vH-plane to the right of line $L_{R\max}^{+}$ which is defined by Eq. (5.1) (Fig. 3). We draw through point O_1 with coordinates H_0, $v_{0A} = 2H_0 \times (a_{A0}\mu_0 + a_{A0}\mu_0')/B_n + v_0$ line L_A which for $0 < H < H_0$ corresponds to combination $AKAR$ and for $H > H_0$ to $AKAS$. The equation of that line is of the form

$$\begin{aligned}
v &= v_0 - \chi(H, H_0) + 2H(a_{A0}\mu_0 + a_{A0}'\mu_0')/B_n, \quad 0 < H < H_0 \\
v &= v_0 - \Psi(H, H_0)(H - H_0) + 2H(a_{A0}\mu_0 + a_{A0}'\mu_0')/B_n, \\
H &> H_0
\end{aligned} \tag{5.3}$$

We draw through point O_1 the lines

$$\begin{aligned}
v_{0A} &= v + \chi(H_0, H), \quad H > H_0 \\
v_{0A} &= v + \Psi(H_0, H)(H_0 - H), \quad 0 < H < H_0
\end{aligned} \tag{5.4}$$

Lines (5.3) and (5.4) divide the region lying to the right of line $L_{R\max}^{+}$ into four parts whose points correspond to the disintegration of an arbitrary discontinuity into the following combination of waves: $RAKAR$, $RAKAS$, $SAKAR$, and $SAKAS$. If the point corresponding to the state O' lies in the lower half-plane $H_0' < 0$, the combination of waves at the disintegration of an arbitrary discontinuity contains one Alfven discontinuity which turns the magnetic field by $180°$ and runs to the right or left of the contact discontinuity. Eight regions which correspond to various combinations of waves and discontinuities and contain one such Alfven discontinuity are shown in Fig. 3 in the lower half-plane $H < 0$. These regions are separated from one another by lines $L_{R\,\max}^{-}$ L_{A1} and L_{A2}. Line $L_{R\,\max}^{-}$ passes through point $v\,(0)$, 0 is defined by the equation

$$v_0 = v + \chi\,(0, -H) + \chi\,(0, H_0), \quad H < 0 \tag{5.5}$$

and corresponds to the disintegration into the combination RKR. To the left (right) of line $L_{R\,\max}^{-}$ lie regions whose points correspond to parameters with subscript $0'$ for which arises a single A-discontinuity running to the left (right) of the contact discontinuity.

Lines L_{A1} and L_{A2} plotted in the lower half-plane vH pass through point $v\,(0)$, 0. Line L_{A1} corresponds to the combination AKR when $-H_0 < H < 0$ and when $H < -H$ to AKS. Line L_{A2} corresponds to combinations KAR and KAS when $-H_0 < H < 0$ and $H < -H_0$ respectively (Fig. 3). The equations of these lines are

$$L_{A1}: \quad v = v_0 - \chi\,(-H, H_0) + 2Ha_{A0}'\mu_0' / B_n, \quad -H_0 < H < 0 \tag{5.6}$$
$$v = v_0 + \Psi\,(-H, H_0)\,(H + H_0) + 2Ha_{A0}'\mu_0' / B_n, \quad H < -H_0$$
$$L_{A2}: \quad v = v_0 - \chi\,(-H, H_0) - 2Ha_{A0}\mu_0 / B_n, \quad -H_0 < H < 0 \tag{5.7}$$
$$v = v_0 + \Psi\,(-H, H_0)\,(H + H_0) - 2Ha_{A0}\mu_0 / B_n, \quad H < -H_0$$

Curves which relate to wave combinations RAK and SAK (Fig. 3) issue from point O_2 of line L_{A1} at coordinates $v_{0A1} = v_0 - 2a_{A0}'\mu_0'H_0 / B_n, H_0$. The equations of these curves are, respectively, of the form

$$\begin{aligned} v_{0A1} &= v + \chi\,(H_0, -H), \quad H < -H_0 \\ v_{0A1} &= v - \Psi\,(H_0, -H)\,(H_0 + H), \quad -H_0 < H < 0 \end{aligned} \tag{5.8}$$

Similarly, point O_3 of line L_{A2} at coordinates $-H_0$, $v_{0A2} = v_0 + 2a_{A0}\mu_0 H_0 / B_n$ is the origin of curves which relate to combinations RKA and SKA. The equations of these curves are, respectively, of the form

$$\begin{aligned} v_{0A2} &= v + \chi\,(H_0, -H), \quad H < -H_0 \\ v_{0A2} &= v + \Psi\,(H_0, -H)\,(-H_0 - H), \quad -H_0 < H < 0 \end{aligned} \tag{5.9}$$

Lines (5.5) - (5.9) divide the lower half-plane into eight regions that correspond to the following wave combinations $SAKS$, $SAKR$, $RAKR$, $RAKS$, $SKAS$, $SKAR$, $RKAS$, and $RKAR$ (Fig. 3). It is seen from Fig. 3 that there exists altogether thirty six possible combinations of waves and discontinuities of the S, R, K and A type when an arbitrary discontinuity disintegrates in the plane case with $H_0 \parallel H_0' \parallel v_0 \parallel v_0'$.

6. Disintegration of an arbitrary discontinuity (the three-dimensional case). Let us consider the three-dimensional problem of disintegration of an arbitrary discontinuity in which the vectors of velocity and magnetic field lie in different planes on both sides of the discontinuity plane. The conditions at the contact discontinuity cannot be satisfied without the introduction of three-dimensional Alfven discontinuities.

We construct the solution of the problem in the plane $\Delta v = v_0 - v_0'$, $\Delta w = w_0 - w_0'$ of the differences of velocity projections on the y - and z - axes to both sides of the discontinuity. In the $\Delta v \Delta w$ -plane the combinations of two S- or R-waves propagating in various directions from the initial discontinuity are represented by regions, while combinations of less than two S- or R- waves are represented by region boundaries in the form of circles.

Let us assume that the initial conditions are such that the three-dimensional initial discontinuity disintegrates into the $AKAS$ combination. In the $\Delta v \Delta w$ -plane the equation of the line corresponding to that combination is

$$(\Delta \mathbf{v}_\tau - \mathbf{L})^2 = \mathbf{R}^2, \quad \mathbf{R} = \frac{-|\mathbf{H}_0'|\,\mathbf{H}_2'}{B_n|\mathbf{H}_2'|}(a_{A0}\mu_0 + a'_{A0}\mu_0') \tag{6.1}$$

$$\mathbf{L} = -a_{A0}\mu_0\frac{\mathbf{H}_0'}{B_n} - a'_{A0}\mu_0'\frac{|\mathbf{H}_0'|\,\mathbf{H}_0}{B_n|\mathbf{H}_0|} + \Psi(|\mathbf{H}_0|, \mathbf{H}_0'|)(|\mathbf{H}_0'| - |\mathbf{H}_0|)\frac{\mathbf{H}_0}{|\mathbf{H}_0|}$$

where the subscript τ at $\mathbf{H}_\tau$ is omitted.

The line defined by Eq. (6. 1) is a circle whose center is at point $\Delta v = L_y$, $\Delta w = L_z$ and radius is equal $|\mathbf{R}|$.

Let $\mathbf{H}_{\tau 0} \parallel \mathbf{H}_{\tau 0}'$. We select the direction of the magnetic field as the y -axis and make the z -axis perpendicular to the y -axis and to the normal discontinuity surface. The center of circle (6. 1) lies on the axis $\Delta w = 0$ so that circle intersects the axis $\Delta w = 0$ at two points Δv_1 and Δv_2. One of these corresponds to the KS -combination and the other to the $AKAS$ -combination, where the magnetic field is turned by $180°$ in the A -discontinuities. Points Δv_1 and Δv_2 lie at distance $|\mathbf{R}|$ from point $\Delta v = L_y$.

Thus for $\mathbf{H}_{\tau 0} \parallel \mathbf{H}_{\tau 0}'$ the line $AKAS$ can be determined as follows. The line $H = H_0'$ is drawn in the vH -plane (Fig. 3) ; its intersection points with the lines that correspond in that plane to combinations KS and $AKAS$ are plotted on the Δv -axis in the Δv, Δw -plane, and these points are rotated about the center. The described line divides the two regions $SAKAS$ and $RAKAS$.

If the magnetic field vector to the right of $\mathbf{H}_{\tau 0}$ is not parallel to the magnetic field vector to the left of $\mathbf{H}_{\tau 0}'$, then $L_z \neq 0$ and the radius of circle (6. 1) remains unchanged. Hence in this case the line to which in the $\Delta v\, \Delta w$ -plane corresponds to the $AKAS$ -combination is a circle drawn on the assumption that $\mathbf{H}_{\tau 0}$, $\mathbf{H}_{\tau 0}'$ and shifted in accordance with Eq. (6. 1).

Let us consider the case of $|\mathbf{H}_{\tau 0}'| > |\mathbf{H}_{\tau 0}|$ with an arbitrary angle between $\mathbf{H}_{\tau 0}$ and $\mathbf{H}_{\tau 0}'$. In that case the circles in the $\Delta v\, \Delta w$ -plane, which correspond to the $AKAS$ - and $RAKA$ -combinations, lie one inside the other and divide the entire plane in regions which correspond to combinations $SAKAS$, $RAKAS$, and $RAKAR$ (Fig. 4). The equations of these curves are similar to (6. 1).

When $|\mathbf{H}_{\tau 0}'| < |\mathbf{H}_{\tau 0}|$ it is necessary to plot in the $\Delta v\, \Delta w$ -region the two curves that correspond to combinations $SAKA$ and $AKAR$ and divide the plane into three regions which correspond to combinations $SAKAS$, $SAKAR$, and $RAKAR$ (Fig. 5).

If $|\mathbf{H}_{\tau 0}'| = |\mathbf{H}_{\tau 0}|$, the circle that corresponds to the AKA -combination divides the $\Delta v\, \Delta w$ -plane into two regions : $SAKAS$ and $RAKAR$ (Fig. 6).

Let all parameters of the medium on both sides of the discontinuity plane be specified. Since the components of field $\mathbf{H}_{\tau 0} \parallel \mathbf{H}_{\tau 0}'$ are specified, it is clear for which lines we have to write down the equations, using formulas (2. 9), (3. 8). and (4. 1), in order to construct the related pattern in the $\Delta v\, \Delta w$ -plane. Having constructed the pattern with known Δv and Δw we find the region in which the point with these coordinates lies, i.e. we determine the combination of waves and discontinuities into which the initial discontinuity disintegrates.

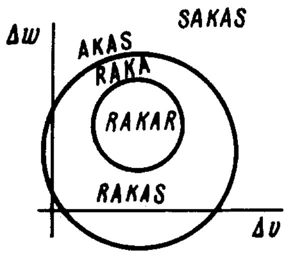

Fig. 4

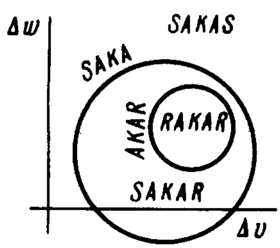

Fig. 5

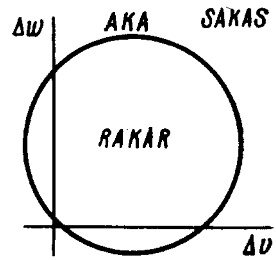

Fig. 6

Equating the sums of jumps of each magnetohydrodynamic quantity at each of the waves and discontinuities generated at the initial jump, we obtain a system of algebraic equations that has to be solved numerically.

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